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DISCREPANCIES WITH Δp SENSORS FOR MONITORING
LIQUID PROPELLANT SLOSHING DURING ROCKET FLIGHT

By Dr. Werner R. Eulitz

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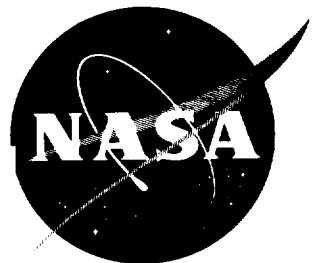This document was prepared for NASA
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ABSTRACT

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This study was made to investigate the suitability of Δp sensors for monitoring liquid rocket propellant sloshing during flight. A generalized amplitude equation has been derived that is applicable to any liquid level. It was realized that a conversion factor was essential for translation of subsurface amplitude measurements into surface amplitude characteristics. Thus, the conversion factor is the ratio of the amplitudes of two unequal levels of the same liquid. The reciprocal of the conversion factor represents the sensitivity of the Δp system at different liquid levels.

The calculated conversion factor and the sensitivity of the Δp system indicate:

    a. Δp sensitivity is a function of liquid frequency.

    b. Δp sensitivity decreases rapidly with increase of depth.

Because the theory developed in this study agrees satisfactorily with test results, it must be concluded that Δp sensors are not suitable for monitoring of liquid rocket propellant sloshing during flight.

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PROPULSION AND MECHANICS BRANCH
PROPULSION AND VEHICLE ENGINEERING DIVISION

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LIST OF ILLUSTRATIONS

DEFINITION OF SYMBOLS

SYMBOL	DEFINITION
ζ, ζ_o	Amplitude of the liquid surface
ζ_z	Amplitude at liquid level z
θ, r, z	Container-fixed cylindrical coordinates
ω	Exciting frequency
ω_{1m}; ω_{11}; ω_n	Natural frequency of the liquid
a	Radius of the container
d =' 2a	Diameter of the container
x_o	Exciting amplitude
g_o	Acceleration due to gravity
$I_1(\zeta)$	Bessel function
a_{1m}; a_{11}; a	Zero's of $I_1'(\zeta)$
p	Pressure
$\phi(r, \theta, z, t)$	Flow velocity potential
h	Filling height of the liquid in the container
ρ	Density
$\lambda = \dfrac{\omega^2}{\omega^2 n}$	Frequency ratio
$r_c = \dfrac{\cosh\left[2a \dfrac{(h - z)}{d}\right]}{\cosh 2a \dfrac{h}{d}}$	

DEFINITION OF SYMBOLS (Concluded)

SYMBOL   DEFINITION

$$\lambda^* = \frac{2\lambda}{(a - 1)(1 - \lambda)}$$

γ = Conversion factor

σ = Sensitivity of Δp measurements

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SUMMARY

This study was made to investigate the suitability of Δp sensors
for monitoring liquid rocket propellant sloshing during flight. A
generalized amplitude equation has been derived that is applicable to
any liquid level. It was realized that a conversion factor was essential
for translation of subsurface amplitude measurements into surface
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of the amplitudes of two unequal levels of the same liquid. The
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The calculated conversion factor and the sensitivity of the Δp
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    a. Δp sensitivity is a function of liquid frequency.

    b. Δp sensitivity decreases rapidly with increase of depth.

Because the theory developed in this study agrees satisfactorily with
test results, it must be concluded that Δp sensors are not suitable for
monitoring of liquid rocket propellant sloshing during flight.

INTRODUCTION

Differential pressure probes sensing the surface amplitude of
liquid rocket propellant sloshing during flight are usually located in

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the plane of motion near the tank wall deep below the surface of the liquid to insure that the monitoring occurs during a large part of the propellant-to-engine drainage period.

Pressure amplitudes decrease with depth of Δp probes. Thus, significance of the data is difficult to determine because the mass of liquid above the Δp probes decreases continuously as propellant is consumed. It is believed that no satisfactory attempt has been made to define the correlation of various subsurface amplitudes with surface amplitudes.

This report is intended to clarify the problem of slosh measurement with differential pressure methods. The work herein is based on the results of the linearized theory, considering maximum liquid amplitude at the tank wall during the first mode of resonance only. Some model tests were made to reinforce the theory.

Acknowledgment

DERIVATION OF THE PRESSURE AMPLITUDE
AT DIFFERENT LIQUID LEVELS DURING SLOSH MOTION

The surface of a liquid in a cylindrical container which is excited by a frequency ω and an amplitude x_0 is described by equation 2.37 in reference 1 as the result of the linearized theory:

$$\zeta = \frac{b}{g} a \cos\theta \, e^{i\omega t} \left[\frac{r}{a} + 2 \sum_{m=1}^{\infty} \frac{I_1\left(a_{1m}\frac{r}{a}\right)}{(a_{1m}^2 - 1) I_1(a_{1m}) \left(\frac{\omega_{1m}^2}{\omega^2} - 1\right)} \right]$$

$$(1)$$

where ζ is the amplitude of any point of the liquid surface with polar coordinates (θ, r) at any instant ωt, and where $b = x_0 \omega^2$. Since the maximum amplitudes $(e^{i\omega t} = 1)$ near the tank wall

($r = a$) in the plane of motion ($\theta = 0$) for the first mode of liquid resonance ($\omega_{11} = \omega_n$ = first natural frequency of the liquid) are of primary interest (ref. 1 and 2), equation (1) simplifies (see Appendix III)

$$\zeta = \frac{b}{g} a \left[1 + \frac{2}{(a^2 - 1)\left(\frac{\omega^2_n}{\omega^2} - 1\right)} \right]$$
(1a)

The pressure p at different liquid levels z below the liquid surface is given by equation 2.36c in reference 1:

$$p = \rho \left(br \cos \theta\, e^{i\omega t} - \phi_t - gz \right)$$
(2)

Dividing by ρg (density and longitudinal acceleration), observing the same condition as above ($\theta = 0$, $r = a$, and $e^{i\omega t} = 1$) for the maximum pressure, and bearing in mind that z is negative for levels below the liquid surface (since the zero point of the tank fixed coordinate system, x, y, z, is supposed to be located in the center of the liquid surface) equation (2) may be transformed

$$\left(\frac{p}{\rho g} - z\right) = \frac{ba}{g} - \frac{\phi_t}{g}$$
(2a)

The flow velocity potential ϕ is defined by equation 2.36a in reference 2:

$$\phi(r, \theta, z, t) = \frac{2i}{\omega} bae^{i\omega t} \cos \theta \sum_{m=1}^{\infty} \frac{I_1\left(a_{1m}\frac{r}{a}\right) \cosh\left[a_{1m}\frac{(h+z)}{a}\right]}{(a_{1m}^2 - 1) I_1(a_{1m})\left(\frac{\omega_{1m}^2}{\omega^2} - 1\right) \cosh\left(a_{1m}\frac{h}{a}\right)}$$
(3)

h = filling height of the liquid above tank bottom. Considering the assumptions above: r = a, θ = 0, and m = 1 (first natural frequency), thus a_{11} = a = 1.84, and ω_{11}^2 = ω^2_n, equation 3 simplifies

$$\phi(a, o, z, t) = \frac{i}{\omega} \, bae^{i\omega t} \, \frac{2}{(a^2 - 1)\left(\frac{\omega^2_n}{\omega^2} - 1\right)} \cdot \frac{\cosh\left[a\,\frac{(h-z)}{a}\right]}{\cosh\left(a\,\frac{h}{a}\right)}$$

(3a)

(z again is negative)

The first derivative of $\phi(a, o, z, t)$ is

$$\phi_t = -\,bae^{i\omega t} \, \frac{2}{(a^2 - 1)\left(\frac{\omega^2_n}{\omega^2} - 1\right)} \cdot \frac{\cosh\left[a\,\frac{(h-z)}{a}\right]}{\cosh\left(a\,\frac{h}{a}\right)}$$

(3b)

Eliminating the term

$$\frac{2}{(a^2 - 1)\left(\frac{\omega^2_n}{\omega^2} - 1\right)}$$

by means of equation (1a), equation (3b) becomes

$$\phi_t = -\,\zeta g \, \frac{\cosh\left(a\,\frac{h}{a}\right)}{\cosh\left[a\,\frac{(h-z)}{a}\right]} + bae^{i\omega t} \, \frac{\cosh\left(a\,\frac{h}{a}\right)}{\cosh\left[a\,\frac{(h-z)}{a}\right]}$$

(3c)

Substituting this in equation (2a), considering the maximum amplitude of motion ($e^{i\omega t} = 1$) and resolving for ζ,

$$\zeta = \left(\frac{p}{\rho g} - z \right) \frac{\cosh \left(a \frac{h}{a} \right)}{\cosh \left[a \frac{(h - z)}{a} \right]} - \frac{ba}{g} \left(\frac{\cosh \left(a \frac{h}{a} \right)}{\cosh \, a \frac{(h - z)}{a}} - 1 \right)$$

(4)

This equation represents the correlation between the liquid surface amplitudes and the pressure amplitude at any level z.

Resolving equation (4) for the pressure amplitude at depth z we obtain

$$\left(\frac{p}{\rho g} - z \right) = \zeta \frac{\cosh \left[a \frac{(h - z)}{a} \right]}{\cosh \left(a \frac{h}{a} \right)} + \frac{ba}{g} \left(1 - \frac{\cosh \left[a \frac{(h - z)}{a} \right]}{\cosh \left(a \frac{h}{a} \right)} \right)$$

(4a)

DISCUSSION OF THE PRESSURE-EQUATION

The hyperbolic cosine-ratio in equation (4a) evidently contains all requisite parameters of the differential-pressure measurements: (1) the submerging depth z of the pressure probes, (2) the height h of the liquid in the tank, and (3) the tank diameter d expressed by $a = \frac{d}{2}$ in equation (4a). This cosine-ratio may be designated

$$r_c = \frac{\cosh \left[a \frac{(h - z)}{a} \right]}{\cosh \left(a \frac{h}{a} \right)} = \frac{\cosh \left[2a \left(\frac{h}{d} - \frac{z}{d} \right) \right]}{\cosh \left(2a \frac{h}{d} \right)}$$

By transformation (see Appendix I) and assuming $h \geq d$

$$r_c = 0.025^{\frac{z}{d}}$$

(4b)

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On the other hand, under the same assumption $h \geq d$ (see Appendix II)

$$\frac{ba}{g} = x_0 \, a \, \lambda$$

Where $\lambda = \dfrac{\omega^2}{\omega_n^2}$

Then equation (4a) may be rewritten

$$\left(\frac{p}{\rho g} - z\right) = \xi \, r_c + x_0 \, a \, \lambda \, (1 - r_c) \tag{4c}$$

The surface equation (1a) can be simplified according to equation (6) of reference 2 and Appendix III

$$\frac{\zeta}{x_0} = a \lambda \left[1 + \frac{2\lambda}{(a^2 - 1)(1 - \lambda)} \right] \tag{5a}$$

Substituting this into equation (4c), the pressure amplitude at depth z becomes

$$\frac{\left(\dfrac{p}{\rho g} - z\right)}{x_0} = a \lambda \left[1 + \frac{2\lambda}{(a^2 - 1)(1 - \lambda)} \, r_c \right] \tag{5b}$$

This equation resembles equation (5a) with the exception of the factor r_c. If z equals zero, which applies to the liquid surface itself, then $r_c = 1$ (see equation (4b)), and equation (5b) is identical to equation (5a)

$$\left(\frac{p}{\rho g} - z\right) \Bigg|_{z = 0} = \zeta$$

Thus, equation (5b) may be considered a generalization of the surface equation (5a). The term $\left(\dfrac{p}{\rho g} - z\right)$, obviously, indicates the liquid amplitude at any level z, where $\dfrac{p}{\rho g}$ is the total pressure minimized by the static pressure imposed by z. Consequently, equations (5a) and (5b) may be combined to make one equation,

$$\frac{\zeta_z}{x_o} = a\lambda \left[1 + \frac{2\lambda}{(a^2 - 1)(1 - \lambda)} \cdot r_c\right] =$$

$$a\lambda \left[1 + \frac{2\lambda}{(a^2 - 1)(1 - \lambda)} \cdot 0.025^{\frac{z}{d}}\right] \tag{6}$$

where ζ_z represents the liquid amplitude at any depth z. If $z = 0$ then $0.025^{\frac{z}{d}} = 1$ and the surface equation becomes identical to equation (5a)

$$\frac{\zeta_o}{x_o} = a\lambda \left[1 + \frac{2\lambda}{(a^2 - 1)(1 - \lambda)}\right] \tag{6a}$$

Equations (6) and (6a) hold under the following assumptions for a cylindrical container of any size and for any acceleration during rocket flight: (1) $\lambda < 1$ (first natural frequency of the liquid), (2) $h > d$, thus (3) $\kappa = \tanh 2a\dfrac{h}{d} = 1$. Due to these assumptions the factor κ may be neglected in the following considerations.

THE CONVERSION FACTOR AND THE SENSITIVITY OF ΔP MEASUREMENTS

From equation (6) it follows that the amplitude of a liquid in motion is different at measurement at different depths. The conversion factor γ from a measured amplitude at probe depth z

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to the actual amplitude of the liquid surface is obtained by dividing the surface equation (6a) by the general equation (6):

$$\gamma = \frac{\zeta_o}{\zeta_z} = \frac{1 + \dfrac{2\lambda}{(a^2 - 1)(1 - \lambda)}}{1 + \dfrac{2\lambda}{(a^2 - 1)(1 - \lambda)} \cdot 0.025 \frac{z}{d}}$$

Designating the term

$$\frac{2\lambda}{(a^2 - 1)(1 - \lambda)} = \lambda*$$

the conversion factor becomes

$$\gamma = \frac{1 + \lambda*}{1 + \lambda* \cdot 0.025 \frac{z}{d}} = \frac{1 + \lambda*}{1 + \lambda* r_c} \tag{7}$$

The reciprocal of γ represents the sensitivity σ of the ΔP measurement dependent on probe depth z. Thus,

$$\sigma = \frac{1 + \lambda* \cdot 0.025 \frac{z}{d}}{1 + \lambda*} = \frac{1 + \lambda* r_c}{1 + \lambda*} \tag{8}$$

represents the percentage of the Δp measured amplitude at depth z of the actual surface amplitude.

In FIG 1, the conversion factor γ is depicted versus λ for different r_c-values or probe-depth z. This figure shows two important facts: (1) the dependence γ on the frequency (ratio λ) and (2) the rapid increase of γ with increasing depth z of Δp measurement.

$$\gamma = \frac{\zeta_o}{\zeta_z} = \frac{1 + \lambda*}{1 + \lambda* r_c}$$

$$\lambda* = \frac{2\lambda}{(a^2 - 1)(1 - \lambda)}$$

$$r_c = e^{-2a\frac{z}{d}}$$

$r_c = 0 \quad z \geq 2d$

$r_c = 0.2 \quad z \approx 0.5\,d$

$r_c = 0.4 \quad z \approx 0.25\,d$

$r_c = 0.6 \quad z \approx 0.15\,d$

$r_c = 0.8 \quad z \approx 0.05\,d$

$r_c = 1.0 \quad z = 0\,d$

$\gamma = \frac{\zeta_o}{\zeta_z}$

λ

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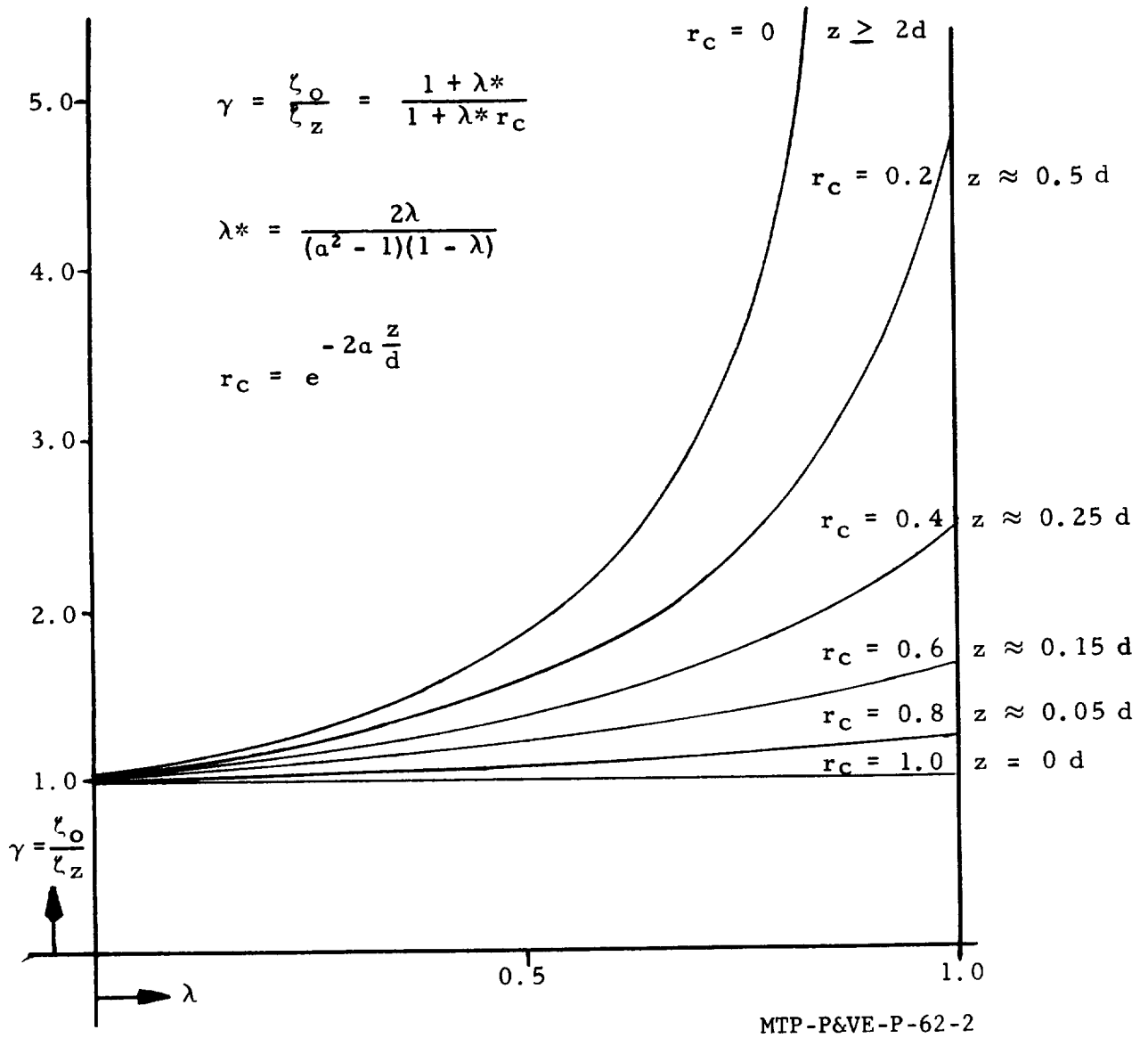
FIGURE 1. CONVERSION FACTOR $\gamma = \frac{\zeta_o}{\zeta_z}$ FOR DIFFERENT Δp PROBE
LOCATIONS z VERSUS FREQUENCY RATIO λ

At the liquid surface ($r_c = 1$; $z = 0$) the conversion factor $\gamma = 1$ for all frequencies (FIG 1). This means, Δp measurements with the probes located at the surface level would indicate the actual magnitude of the liquid surface amplitude. It was found experimentally during earlier investigations (ref. 3) that at a level $z \approx 0.25\,d$, the conversion factor γ at high slosh frequencies ($\lambda > 0.75$) is roughly 2. If the Δp probes are located at a level $z > 2d$, the conversion factor for higher slosh frequencies increases rapidly which means the recorded Δp amplitude is very small in comparison to the actual surface amplitude.

In FIG 2 the sensitivity σ (Δp amplitudes in percent of surface amplitudes) is depicted versus depth ratio z/d. At the liquid surface ($r_c = 1$; $z = 0$) the sensitivity σ of the Δp probes is 100 percent. At a level $z = 0.25\,d$ below the surface, the sensitivity at high frequencies ($\lambda > 0.75$) is roughly 50 percent; and at a level of $z > 2d$ the sensitivity σ of Δp measurements decreases to 20 percent, becoming rapidly lower at high slosh frequencies (large liquid amplitudes at the surface). FIG 1 and 2 reveal that the sensitivity of Δp measurements as well as the conversion factor are more reliable at low frequencies of the tank motion. This is further illustrated in the graph of FIG 3. Here, the dimensionless amplitudes ζ_z/x_0 are depicted versus the dimensionless depth z/d for three different λ-values. At low frequencies of tank motion ($\lambda = 0.1$), the amplitudes indicated by Δp measurements do not change considerably with increasing depth of Δp probe location (z/d). At high frequencies ($\lambda = 0.9$)? however, the Δp amplitudes decrease rapidly with increasing depth, and at a level of about 1.5 tank diameters below the surface, the Δp measured amplitudes are practically constant and relatively small even at high frequencies.

This also follows from equation (6). If z is large, then the term

$$\frac{2\lambda}{(a - 1)(1 - \lambda)} \cdot 0.025^{\frac{z}{d}} = \lambda * r_c \approx 0$$
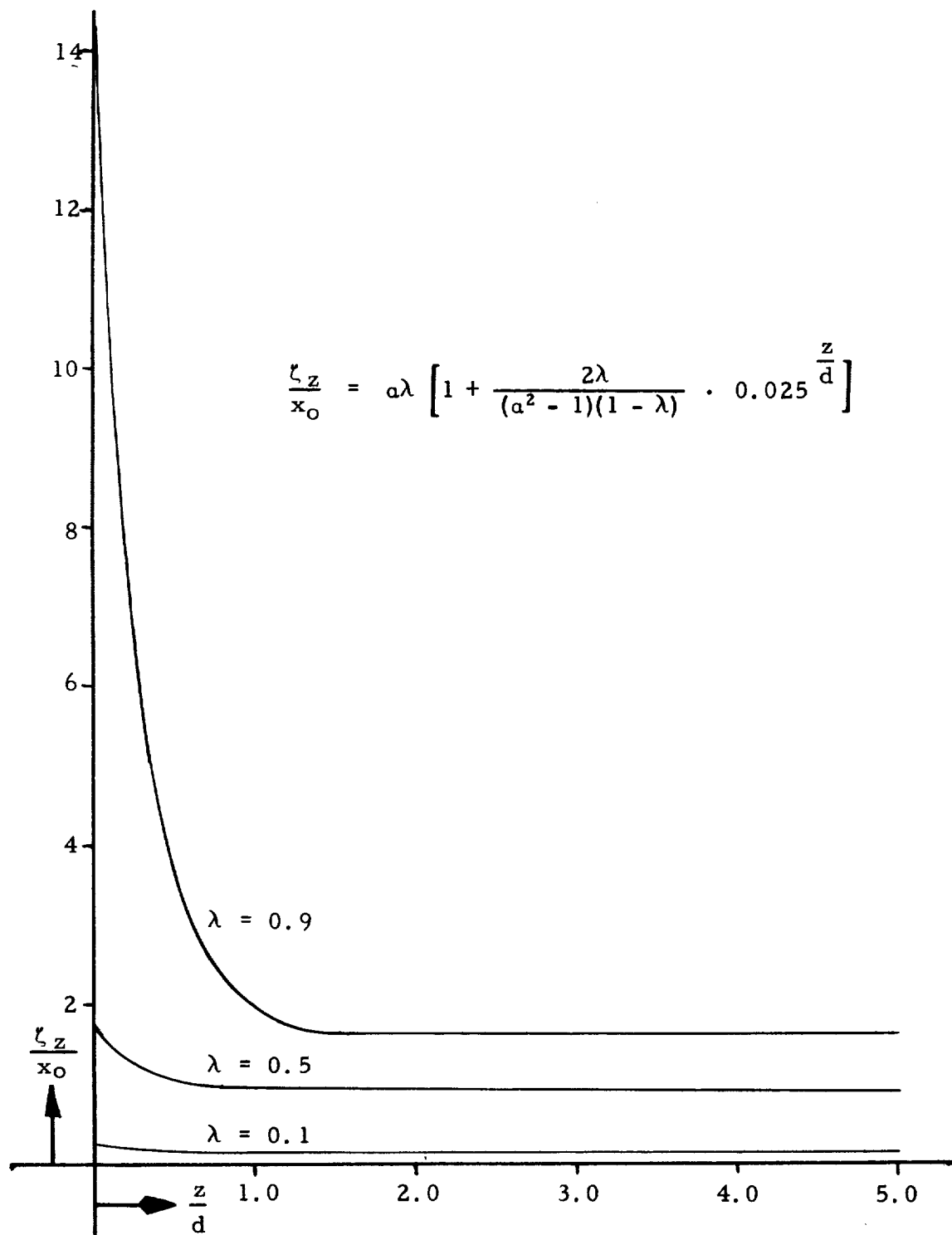
and the amplitude in dimensionless form

$$\frac{\zeta_0}{x_0} = a\,\lambda = \text{constant}$$
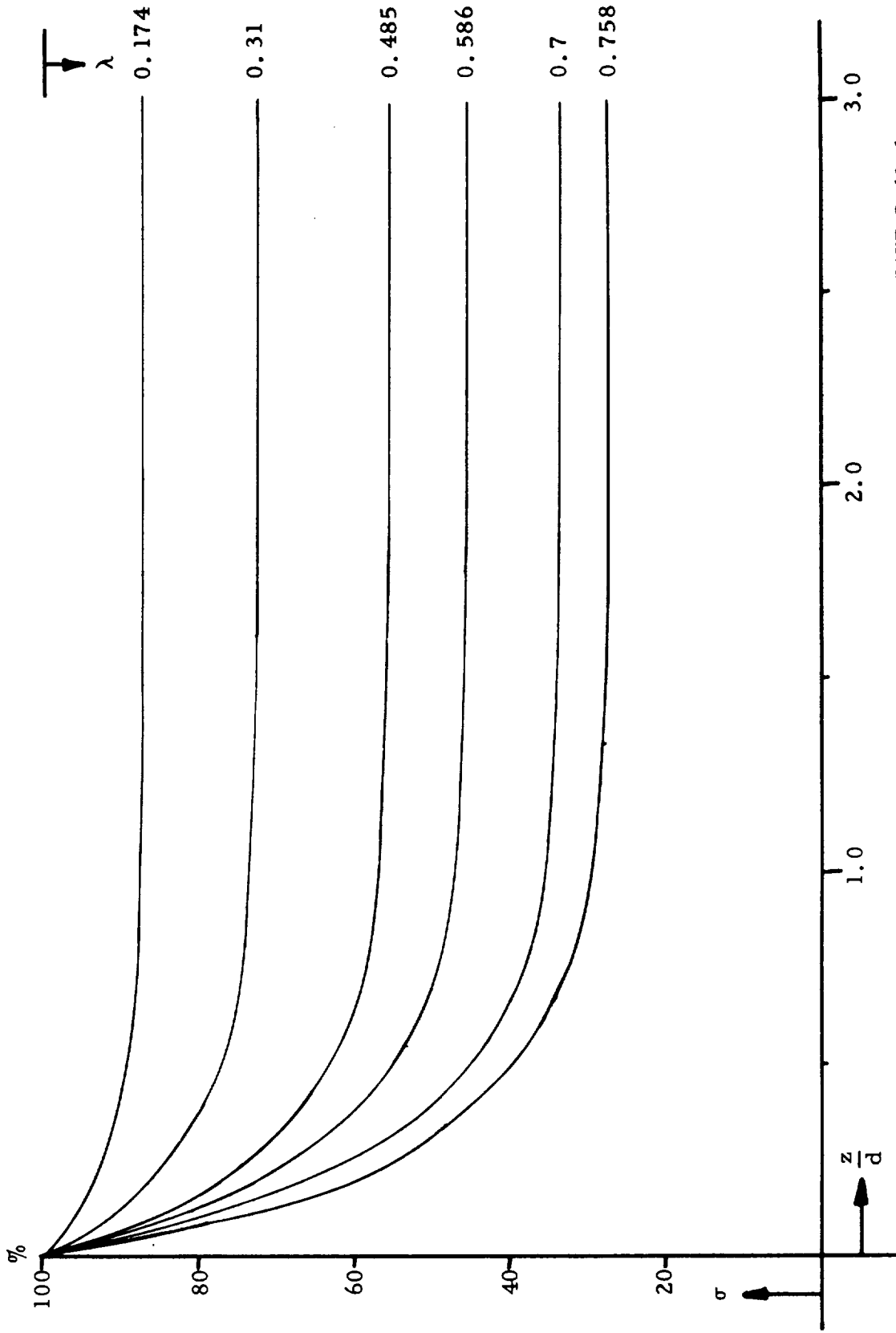
FIGURE 2. SENSITIVITY σ OF Δp AMPLITUDE MEASUREMENTS IN PERCENT OF LIQUID
SURFACE AMPLITUDE ζ_o VERSUS DEPTH RATIO z/d
FOR DIFFERENT FREQUENCY RATIOS λ

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FIGURE 3. PRESSURE AMPLITUDE AT DIFFERENT LEVELS
FOR DIFFERENT λ

for a particular λ. Thus, the maximum amplitude ζ_z/x_0 resulting from Δp measurements deeper than 1.5 tank diameters below the surface cannot exceed the value $a = 1.84$ for $\lambda = 1$ (first natural frequency of the liquid). Consequently, if the amplitude of container movement is $x_0 = 0.5$ inch at the level of a Δp probe located more than 1.5 tank diameters below the liquid surface, the actual amplitude at resonance ($\lambda = 1$, very high slosh motion at the surface) indicated by the Δp measuring system is only $\zeta_z > 1.5d = x_0 \, a \, \lambda = 0.92$ inch. Actually, this amplitude will be even smaller, since the maximum amplitude always occurs at $\lambda < 1$ due to the slosh condition

$\zeta_0 = \dfrac{g}{\omega^2}$ (see refs. 1 and 2). The consequences may be illustrated by a practical example.

Suppose a 70-inch-diameter container filled with liquid oscillates laterally with an amplitude $x_0 = 0.5$ inch. The maximum liquid surface amplitude would occur at a frequency ratio $\lambda = \omega^2/\omega^2_n \approx 0.96$ and it would amount to about 20 inches. The Δp probes located about 140 inches (2 tank diameters) below the surface would record an amplitude of $\zeta_{140} = x_0 \, a \, \lambda \approx 0.88$ inch. This is, roughly, only 4.5 percent of the actual surface amplitude. If the amplitude of tank motion would increase to $x_0 = 1$ inch, the maximum amplitude at the liquid surface would increase to about 21 inches. But in this case, the maximum surface amplitude would occur at $\lambda \approx 0.92$ and the Δp probe would record an amplitude of $\zeta_{140} = x_0 \, a \, \lambda \approx 1.7$ inches, or roughly 8 percent of the actual surface amplitude. This example demonstrates that the amplitudes recorded by a Δp system deep below the liquid surface are largely dependent upon the magnitude of the exciting amplitude x_0 at this particular level while the maximum surface amplitudes do not change considerably. Thus, the same large surface amplitude can be recorded differently by the Δp measuring system.

From this theory it follows that Δp measurements made at great depths to monitor liquid propellant sloshing during rocket flight are highly unsuitable. Thus, conclusions occasionally drawn from rocket flight test data that sloshing occurred not earlier than, say, after 100 seconds of flight time, cannot be considered correct. The fact is that the pressure probes located deep below the surface probably did not respond properly to the heavy surface motion during the first period of flight. The above considerations indicate (FIG 3) that Δp measurements down to only $\frac{1}{2}$-diameter depth can be considered fairly reliable.

EXPERIMENTAL PROOF OF THE THEORY

The fundamental amplitude equation (6) refers to the amplitude on one side near the tank wall in the plane of motion; while, in practice, two probes are located at equal levels on opposite sides of the container, thus actually measuring the double amplitude. In order to eliminate the influence of the opposite probe, a test setup has been built with one probe of the Δp system located in a tank of 17.5-inch diameter oscillating around a pivot point, and the other probe located in an adjacent stationary tank. Both tanks were filled with water to equal levels and the probes were also submerged to equal levels, thus eliminating the depth z of the probe levels and measuring the amplitude at the probe level by the probe in the oscillating tank only, according to equation (6). The tanks were filled up to about 3 diameters of the oscillating tank and the probes were lowered in steps of 5 inches down to 50 inches, starting at a depth of one-quarter diameter of the oscillating container.

The 17.5-inch container was oscillated pendulum-like with an amplitude of x_o = 0.5 inch by a driving shaft acting on the system beneath the bottom of the tank about 110 inches below the pivot point. The tank was oscillated with increasing frequencies from 0.5 to 1.3 cps, the first natural frequency of the liquid at this tank size being 1.435 cps. This frequency sequence was exercised in two test runs for each probe location to obtain better average results.

Referring to the fundamental amplitude equation (6) the test data also were converted into dimensionless terms. From the frequencies ω applied to the oscillating container system and the knowledge of the first natural frequency of the liquid system ω_n the λ-values for each test condition were determined $\left(\lambda = \dfrac{\omega^2}{\omega^2_n} \right)$. The depth of the Δp probes z was expressed in terms of the tank diameter (z/d), and the measured amplitudes ζ_z at different Δp probe positions z were divided by the acting amplitude x_o at each particular probe level which evidently varies from level to level due to the pendulum-like motion of the tank.

As in FIG 3, the dimensionless amplitudes ζ_z/x_o calculated for the λ-values resulting from the test frequencies were also plotted versus the z/d ratio according to equation (6). The experimental λ-values are indicated at the right hand side of the graph (FIG 4). The
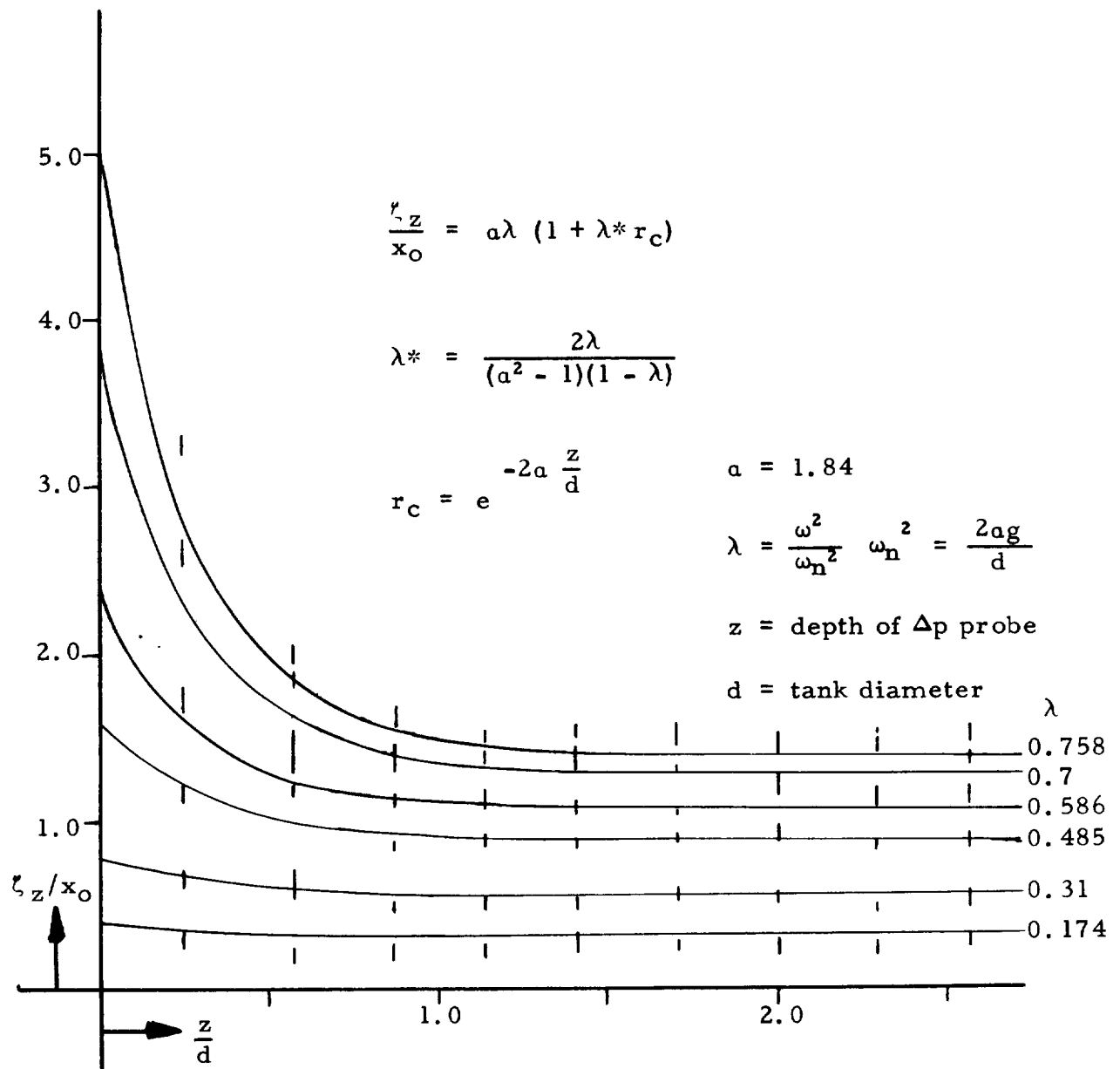
FIGURE 4. LIQUID AMPLITUDES AT EXPERIMENTAL λ-VALUES
VERSUS z/d. SOLID CURVES THEORETICAL, VERTICAL MARKERS
EXPERIMENTAL RESULTS, THE LENGTH OF WHICH
INDICATES THE SPREAD

experimental dimensionless amplitudes ζ_z/x_0 are depicted by vertical markers for each Δp probe position (z/d). The varying length of the markers indicates the spreading of the test results.

FIG 4 impressively shows the agreement of the theory with actual experience. It also proves again, as discussed previously, that Δp measurements for monitoring surface sloshing are fairly suitable down to depths of, at most, one-half of the tank diameter. Slosh measurements with Δp probes located far below the surface, however, are absolutely undependable. This is aggravated by the fact that the Δp probes are highly sensitive; there is no way to insulate them against odd shocks caused by an external force or vibrations of the tank wall or the probe pipelines, and these shocks will be recorded by the Δp system also. As it has been previously demonstrated, these odd vibrations occasionally occurring at the probe level can overshadow the pressure variations due to surface sloshing in such a way that evaluations of flight data and dependable conclusions about the slosh conditions of the liquid surface are impossible.

Recent tests with lateral tank motion (x_0 equal at all levels) show the same result directly by the Δp amplitudes dropping with depth of measurement. Thus, the application of the Δp measuring system to monitor propellant slosh under flight conditions not only involves the risk of erroneous conclusions, it also hazards wasting time and money. By determination of the essential parameters during flight (the frequency ω of the container, the longitudinal acceleration g of the rocket, the tank diameter d, and the momentary displacement x_0 of the liquid surface center), the actual liquid surface amplitudes can be calculated to much greater approximation than by evaluation of Δp measurements. If anti-slosh baffles are installed in the container, the liquid amplitudes are disturbed anyway, and discussions on liquid amplitudes are meaningless under these conditions - where the linearized theory is not valid (see ref. 1 where the limits of the linearized theory were discussed).

APPENDIX I

$$r_c = \frac{\cosh\left[a\,\frac{(h-z)}{a}\right]}{\cosh\left(a\,\frac{h}{a}\right)} = \frac{\cosh\left[2a\left(\frac{h}{d} - \frac{z}{d}\right)\right]}{\cosh\left(2a\,\frac{h}{d}\right)}$$

$$r_c = \frac{\cosh\left(2a\,\frac{h}{d}\right)\,\cosh\left(2a\,\frac{z}{d}\right) - \sinh\left(2a\,\frac{h}{d}\right)\,\sinh\left(2a\,\frac{z}{d}\right)}{\cosh\left(2a\,\frac{h}{d}\right)}$$

$a = 1.84$

If $h \geq d$, then, $\cosh\left(2a\,\frac{h}{d}\right) = \sinh\left(2a\,\frac{h}{d}\right)$, and

$$r_c = \cosh\left(2a\,\frac{z}{d}\right) - \sinh\left(2a\,\frac{z}{d}\right) = e^{-2a\,\frac{z}{d}} = \left(e^{-2a}\right)^{\frac{z}{d}} =$$

$$0.025^{\frac{z}{d}}$$

APPENDIX II

The acceleration b due to forced motion is

$$b = x_o \omega^2$$

The first natural frequency ω_n of the liquid is given by the formula

$$\omega^2_n = \frac{ag}{a} \tanh a \frac{h}{a} = \frac{ag}{a} \kappa \qquad \begin{array}{l} \kappa = \tanh a \dfrac{h}{a} = 1 \text{ for} \\[2mm] h \geq d \text{ (see Appendix I)} \end{array}$$

Thus,

$$\frac{a}{g} = \frac{a}{\omega_n^2}$$

and

$$\frac{ba}{g} = \frac{x_o a \omega^2}{\omega_n^2}$$

Designating $\quad \lambda = \dfrac{\omega^2}{\omega_n^2}$

$$\frac{ba}{g} = x_o a \lambda$$

APPENDIX III

The liquid surface amplitude equation under forced oscillations of the container is according to reference 1:

$$\zeta = \frac{b}{g} a \cos\theta \, e^{i\omega t} \left[\frac{r}{a} + 2 \sum_{m=1}^{\infty} \frac{I_1\left(a_{1m}\frac{r}{a}\right)}{(a_{1m}^2 - 1)\, I_1(a_{1m}) \left(\frac{\omega_{1m}^2}{\omega^2} - 1\right)} \right]$$

Conditions of practical interest:

Maximum amplitude (largest tank displacement) $\quad e^{i\omega t} = 1$

in the plane of motion $(\theta = 0)$ $\qquad\qquad\qquad \cos\theta = 1$

at the tank wall $(r = a)$ $\qquad\qquad\qquad\qquad \dfrac{r}{a} = 1$

Assumptions

First natural frequency of the liquid considered only $(m = 1)$

$$a_{1m} = a_{11} = a = 1.84$$

$$\omega_{1m}^2 = \omega_{11}^2 = \omega_n^{\,2} = \frac{2ag}{d} \tanh 2a\, \frac{h}{d} = \frac{2ag}{d} \quad \text{for } h \geq d$$

$$\frac{\omega^2}{\omega_n^{\,2}} = \lambda$$

Then,

$$\zeta = \frac{b}{g} a \left[1 + \frac{2}{(a^2 - 1)\left(\frac{1}{\lambda} - 1\right)} \right]$$

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According to Appendix II

$$\frac{b}{g} \, a \; = \; x_O \, a \, \lambda$$

Then

$$\zeta \; = \; x_O \, a \, \lambda \left[1 + \frac{2\lambda}{(a^2 - 1)(1 - \lambda)} \right]$$

and

$$\frac{\zeta}{x_O} \; = \; a \, \lambda \left[1 + \frac{2\lambda}{(a^2 - 1)(1 - \lambda)} \right] \qquad\qquad \text{equation (5a)}$$

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APPROVAL   MTP-P&VE-P-62-6

DISCREPANCIES WITH Δp SENSORS FOR MONITORING
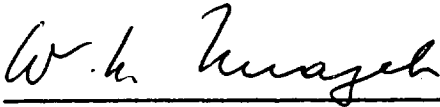LIQUID PROPELLANT SLOSHING DURING ROCKET FLIGHT

By Dr. Werner R. Eulitz

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classification. Review of any information concerning Department of
Defense or Atomic Energy Commission programs has been made by
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